Computational Imaging

- AOP Review
- Introduction:
  - Computational imaging is the joint design of the front-end optics and post-detection signal processing
  - exceeds the physical limit of the optics
  - reduce the requirements in size, weight, power, cost
  - section2: sensing & imaging: delineate definitions between remote sensing, photography, image enhancement and recovery
    section3: history of imaging
    section4: essentials
    section5-7: 3 imaging systems
    section8: assess the computational imaging: strengths & weakness
    section9: summary
- Sensing, Imaging, and Photography:
  - sensing & imaging: sensing doesn't need to reference the spatial map, while imaging needs it
  - two reasons for an image:
    - estimate the spatial map of a physical property
    - classify elements of the environment into categories
  - imaging & photography: photography means to document an event or elicit an emotion, spatial relationship between object point and image point
- Short history of imaging:
  - Antiquity: lack the knowledge and understanding of how to use it
  - Aided human imaging: beginnings of optical science and engineering
  - Photochemical-based recorded imaging & digital-electronic recorded imaging
  - Computational imaging: joint design with the computation
- Computational Imaging:
  - Definition:
    - sensing: locations of the measurement & source information are coincident
    - imaging: are not coincident
    - pre- post-:
      - pre-: use optics to transform incoming wavefronts
      - post-: CT
      - both pre- post-: computational imaging: pre- aims to simplify the information we desire & indirect mesurement of the information
  - Physical and Mathematical Essentials:
    - Schematic representation of an imaging system:
      - image.png
    - coherence:
      spatial coherence: at the same time but from different spatial loacations
      temporal coherence: from the same point at different times
    - illuminating:
      active illuminating: can control the illuminating source
      passive illuminating: self-luminous or is illuminated by an external source
    - consider polychromatic, spatially structured source:
      - image.png
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    - So:
      - CSF = FFT(pupil function)

        - incoherent OTF = 2d autocorrelation(pupil function)   (integrated?)
        - incoherent PSF = FFT(incoherent OTF)
    - consider discrete detectors:
        - image.png
          image.png
          image.png
- Optical design(不懂光学设计，暂时跳过不看)
- Information Essentials
    - Rayleigh resolution limit: $1.22\lambda f$
    - F-number: $F = \frac{f}{D}$
    - SBWP(Spatial Band Width Product): $S = (\frac{W_d D}{\lambda f})^2$ represents the upper bound on information content
    - Noise
        - Optical noise: Possion process / Gaussian Noise with i.i.d (large signals) and we can use large apertures & long detector integration times while reduce the high frequency temporal signals
        - Electronic noise: random generation of electrons within the detector
          if spatial sampling rate is not sufficiently high, aliasing is present in the measurements, additive noise
    - Estimation:
        - The key element of Computational Imaging is designing a measurement X and post-detection processing to estimate a desired θ
        - If the estimator $\hat{\theta}(X)$ is the unbiased estimator for θ, the Cramer-Rao inequality places a lower bound on the estimator uncertainty: $\Delta\hat{\theta}^2 >= \frac{1}{F(\theta)}$ and $F(\theta)$ means Fisher information: image.png
    - Core:
        - The goal in Computational Imaging is to optimize information in each measurement relative to a parameter of interest, which means that one can encode wavefront information in such a way that uncertainty in parameter estimation is minimized after measurement. Thus, benefits can be gained by designing the measurement and post-processing algorithms in unison. Important considerations in the co-design of optics and post-detection processing are the ease with which the encoding can be inverted, its sensitivity to noise, and identifying its associated null spaces.
- Motivations
    - impossible to realize using conventional means
    - address a dimensionality mismatch
    - reduces the cost of making measurements in comparison to conventional means
    - encoding three planes: source, pupil and detectors

- Motivation1: Conventional Imaging is Impossible
    - Phase Imaging: objects may be transparent and modulate only the phase of wave.
        - Radio frequences can measure the phase directly but not for visible and IR frequences. Computational approaches requires a combination of front-end wavefront manipulation and post-detection processing to extract the phase.
        - Phase Contrast Microscope：Proposed by Zernike
            - using an on-axis phase filter in the Fourier plane of the pupil plane so that they can achieve the interference, which can make the background light to dark and improve the contrast
            - while this is an example of optical processing, not the computational imaging
        - Interferometry and Holography: Convert the phase change to the intensity change
            - Holography provides a mean to visualize not just the phase of a scene, but the scene that gave rise to the phase. And the off-axis holography allows one to separate the real images.
        - Phase Retrieval from Magnitude: Concluded as 2 approaches
            - Error-reduction approach: uses a single intensity measurement(typically the Fourier magnitude) and the iterative application of constraints in both the image and Fourier

        domains to generate a Fourier phase that satisfies all the physical constraints.   GS算法?
        - Input-output algorithm: strict object constraints is not required. Instead, one alters the object estimate in a manner that reduces error in the output in the image domain.   SGD?
        - Specification: A prior information and the constraints is key.
          Principal constraint is that the object has finite extent, and its intensity must be nonnegative.
          support constraints, nonnegativity and sparsity
  - Quantum Imaging:
    - Quantum photon entanglement occurs when photons are generated in such a manner that the quantum state of one photon is dependent on the state of the other photons.
    - Coincidence or ghost imaging is done by processing the photon statistics from the two detectors to look for correlations.
    - Thermal ghost images exhibit a large bias, which increases noise and degrades image quality.
  - Volumetric Imaging:
    - Fundamental issue is the inability to image through objects at wavelength in the visible and IR spectrum.
    - The images created from a sequence of direct measurements are not considered as computational imaging.
    - Computed Tomography: 3*10^16Hz~3*10^19Hz
      - The dominant effect on x rays as they traverse through a material is a change in amplitude, not phase, proportional to the density of material encountered.
      - Reconstruction method: back projection algorithm proposed by Johann Radon, which based on the Fourier Slice theory.
    - Magnetic Resonance Imaging:
      - three components: a static magnetic field, a dynamic magnetic field that produces a one-dimensional spatial gradient whose direction varies in time and source to produce a sequence of RF pulses.
      - magnetic fields encode spatial information by setting the resonant frequencies of hydrogen nuclei in the body. Each pulse of RF energy excites the nuclei briefly. The nuclei release the RF energy they absorbed, which is measured by RF detectors. The frequency at which the energy is detected indicates the spatial location from which is was transmitted, and the strength of the detected signal is proportional to density of hydrogen nuclei at that location.
      - post-processing: filtered backprojecion ...
- Motivation2: Dimensionality mismatch: wavelength, depth, polarization
  - plenoptic function: $L(x, y, z, \theta, \phi, \lambda, P)$
  - label imagers as the computational imaging if they measure the parameter indirectly, but not direct for scanning.
  - Spatial-Spectral Imaging: the goal is to produce a data cube with two spatial dimensions and one wavelength dimension as few as possible. NxNyNlambda
    - Designing a single optical system that has both high spatial resolution and high spectral resolution represents a dimensional mismatch
    - Bayer assigned green filters to two pixels because human vision is more sensitive to green than others, while the fiter introduces so-called mosaicking artifacts which is smoothed by the post-processing of the interpolation. However, the filter compress a scene's spectrum into three broad color bands. To increase the frequency resolution, requires spectral dispersion
    - We do not consider the scanning approaches computational such as scanning spectrum or scanning with the spatial dimension.
    - Instead, recent work shows how exploiting spectral dispersion can access information throughout the entire data cube. The approaches essentially implement a polychromatic PSF,

$$i(x, y, \lambda) = |o(x, y, \lambda)|^2 ** |p(x - S_x(\lambda), y - S_y(\lambda))|^2$$

    - CTIS(computed tomography imaging spectrometer) is grounded in the diffractive elements to generate multiple projections through the cube simultaneously. The entire cube can be reconstructed by the computed tomography skills. However, it requires the detector to large enough, which determines the spectral resolution. Actually, CTIS do not encode the information, so that it does not use efficiently the information capacity available of the system.

- CASSI(coded aperture snapshot spectral imaing) & SCCSI(snapshot colored compressive spectral imager)
  - CASSI: spectrum is dispersed by a prism and the dispersed object is coded with a binary amplitude mask, so the deteced image is:

    image.png

    Coding improves the preformance of systems in the presence of the noise. Thus it is robust than before.
  - SCCSI: similar to CASSI except the code spans the spatial and spectral dimensions

    image.png

    image.png

    $M = P\Gamma D|O|^2 + n$

    D: the dispersion by a prism, {Γ} is the spatial-spectral coding matrix, P is the integration matrix that accounts for the spatial convolution of the imaging system.
- Three-dimensional Imaging:
  - It is not possible to image through a volume at optical wavelength, but it is possible to image three-dimensional surface. This section contains two interpretations of three-dimensional imaging: PSF is invariant to depth and one in which the variation in PSF as a function of depth produces a high-resolution measurement of a three-dimensional surface.
  - Depth-invariant imaging:
    - We know the depth of field is related to the f#, if the aperture is small, the DOF will be larger, while reduces light collection and thus, signal information, which decreases the image SNR.
    - If the scenic is static, we can capture multiple images and segment each into regions of best focus, and merge these regions to a single image. However, dynamic scenes increase the motion blur as multiple iamges are captured over time.
    - As a consequence, we can choose to design the PSF or OTF to make it invariant to misfocus over some range of object distances. If the pupil function is:

      image.png

      By using this, the OTF with respect to the misfoucs distance can be relative similar, so that they can restore the image by using the same method, while the shortage of it is that the high frequency of the image will be reduced. Current OTF and traditional OTF are compared below:

      image.png
  - Depth Measurement: we don't consider the tof imaging (just reflection) & proximate measurements of a surface & estimate the depth using visual cues. What we discussed there is the optical techniques for reconstructing a 3-D surface that are based on a forward model that has been engineered to enable the estimation of depth.
    - Structured illumination: active illumination

      - applying a spatial code in the object plane, which aims to match each projected point to a point of the original pattern. This technique is now sufficiently sophisticated.
    - Double-helix PSF:
      - design the PSF to make it unique with respect to the distance, it contains two laterally separated points whose separation and angular orientation relative to the optic axis are functions of depth.
    - Stereoscopic photography:
      - human stereo vision, it needs to match the object points in both images, remarkable effort
    - Integral imaging:
      - use microlens to form multiple images of a single scene from many perspectives, reconstruction is achieved via backprojection of multiple two-dimensional projections. Resolution is determined by each subimage formed by each microlens, depth resolution is is determined by the pair of subimages with the longest baseline, which is a function of the array geometry and size.
    - light-field camera:
      - objective lens + microlens array: encode the directional light-ray so that get the information of the whole light-field, which can reconstruct the field everywhere.
      - The spatial format of the captured and reconstructed image is limited by the size of the microlens array and not the space-bandwidth of the objective or focal plane array.
      - image.png
  - Polarization Imaging: Fields can become polarized upon reflection by or transmission through objects.
    - polarization state of a field is commonly expressed by the [[Stokes Vector]]
    - core idea is the measurement of the light reflected or refracted by the material which can change the polarization of light.
    - concerning the polarization measurement, which can be separated with the space & temporal, correponding to the superpixel & highspeed camera.
- Motivation3: Reducing Measurement Marginal Costs
  - Possible but Impractical:
    - SAR(Synthetic Aperture Radar):
      - resolution is determined by the diffractive limit(Rayleigh) so that is impractical to design the system which satisfy the requirement of the resolution level: 10cm/1km. So SAR
      - collect multiple, spatially displaced measurements using a single physical aperture, and synthesizing a virtual radar aperture from those measurements.
      - image.png

        SAR provides a practical solution to overcome some of the performance limitations of conventional radar, while retaining some of its advantages, including day, night, and all-weather operation.
    - Coded-Aperture Imaging:

            - x-ray & γ-ray, these refractive index of most materials are approximately 1, thus fabricating conventional imaging elements, such as lenses and mirrors. Usually we use the pinhole to substitute the lenses, while limit the long data acquisition times or poor signal-to-noise conditions.
            - As the diffraction of these wavelengths is minimal, the PSF of the system is just the projection of the Mask, not the transform, so:

$$i(x,y) = |o(x,y)|^2 * * P(x,y)$$

$$o_2(x,y) = i(x,y) * * T(x,y) = |o(x,y)|^2 * * [P(x,y) * * T(x,y)]$$

              if $P(x,y) * * T(x,y) \approx \theta(x,y), o_2(x,y) \approx |o(x,y)|^2$, thus, a coded aperture can be used to produce x-ray and γ -ray imagery as effectively as a pinhole with a substantial improvement in acquisition time, SNR, or both.
- Possible but not optimal: conventional methods can solve but not the best solution
    - Active illumination provides control over the imaging environment and the ability to manipulate the appearance of an object, such as medical imaging, diagnostics, quality control, and cooperative security.
      In contrast, passive illumination relies completely on ambient or natural sources of light, such as photography, astronomy, surveillance, and non-cooperative security.
    - Superresolution:, resolution & aperture size
        - structured illumination & production of moire fringes modify the object so that make it can be resolved. Recently, they can also be used to extract the depth information from a scene with high lateral spatial resolution.
        - Iterative methods overcome the physical limitations in passive imaging, but only the marginal results can be obtained due, to noise limitations, while active illumination with multiple images introduces more degrees of freedom, which enables to extract information from a single image that is beyond conventional physical limits.
        - Three ways to increase resolution of the microscopy:
            - change the measurement from one of resolution to localization and uses statistical post-detection processing to generate a PSF whose width is less than the resolution limit set by the microscope's aperture
            - performs a series of source-coded measurements of a scene and uses phase retrieval to combine the measurements coherently
            - In Gustafsson's work, spatially structured illumination is used to achieve lateral resolution with a microscope that significantly exceeds the classical diffraction limit
        - Superresolution:
            - resolution is limited by the Abbe diffraction limit: roughly 250nm

              image.png

            - superresolved fluorescence microscopy(SRFM): by using photon statistics, resolution can be more higher than the Rayleigh's two point criterion
                - SRFM relies on the activation of a sparse subset of molecules at one wavelength (e.g., 405 nm) that are imaged at a second wavelength (e.g., 561 nm) until most of the illuminated molecules are bleached. A different sparse subset of molecules is illuminated to produce a second image, and the process is repeated until the entire sample is inactive.
                - if one assumes each molecule is a point source and that the probability density of photons arriving at the focal plane from that point source is approximately Gaussian

                  image.png
                  The minimum uncertainty in estimating the center (μx, μy) of the PSF p(x, y)scales with the number of photons collected N:

                  image.png
                  Therefore, the error in estimating the centroid location of the molecule is less than the Abbe diffraction limit by a factor of 1/sqrt(N)
                - Extensions of this method such as stochastic optical reconstruction microscopy (STORM), have enabled additional capability and allowed this condition to be relaxed

- Fourier Ptychography Microscope:
  - above design limits the SBWP(resolution and volume?), thus we need to redesign optics, which brings optical aberrations or increases the cost of the optics. if we scan the sample, we also need the precise control of the system.
    FPM assumes that illumination from a single LED creates a plane wave incident on the sample at an angle related to the LED's position in the array. if we assumes the PSF not changed, illumination sources location change means that we measure the sample from different positions of the Fourier plane, then using phase retrieve methods can reconstruct the image.
  - image.png

    1.defines the fourier space of the high-resolution iamge

    $$G_h = \sqrt{(I_h)}e^{i\phi_h}$$

    2.filter fourier region of the current illumination angle:

    $$G_l = \sqrt{I_l}e^{i\phi_l}$$

    3.use the actual intensity measurement to replace the initial amplitude

    $$G_l' = /sqrtI_m e^{i\phi_l}$$

    4.repeat
  - Thus we can acquire a large FOV and a larger SBWP by using the phase retrieval method.
- Increased Temporal Resolution:
  - Motion that translates across multiple pixels in a single exposure will produce smearing or motion blur in the result image for the reason that the long exposure time will cause a sinc-function in the frequency domain which will convolve with the static scene. However, deconvolution in this question is difficult for the ill-conditioned forward model.
  - Solution:
    - use the short exposure time low SNR
    - algorithm based on iterative maximum likelihood estimation methods are useful in the global & uniform. localized
    - oscillate the imager's position to freeze moving objects in a scene don't modify the imaging optics
    - engineer the PSF to remove zeros in the corresponding OTF and preserve high frequencies, the fidelity of reconstruction image is determined by the transfer matrix condition number
- Form Factor Constraints:
  - Lensless On-Chip Microscopy: more compact & less expensive & highly portable systems & FOV & resolution
    - above high resolution & large FOV microscopy needs large volume because of the long focal length of the objective lens which can not satisfy the requirement of the compact package

- Thus lensless on-chip microscopy, it places the sample on or in close proximity to the array and process the diffraction images to obtain the amplitude & phase. However, this relies upon proximate imaging, not computational imaging. And the resolution of it can be improved & limited by using a microlens array.
- If the illuminating resources are coherent, Gabor hologram can be applied to reconstruct the image, the field we get at the plane immediately below the sample is:

$$E_sample(x,y) = A_{ref}(x,y) + A_scat(x,y)e^{j\phi_{scat}(x,y)}$$

The hologram on the detector array is a convolution of it with a free space propagation kernel:

$$E_{meas}(x,y) = |E_{sample}(x,y) * *h(x,y)|^2$$

- Multi-Aperture Imaging:
  - two groups:
    - an array of apertures and a single detector array
    - pair each aperture to a separate detector array----------multi-camera system
      - by mapping individual aperture to other parameters, it can be used to dynamic imaging, polarimetric imaging and multispectral imaging.
  - imager's performance is related to the geometry, thus if we change the size to improve resolution of optics, which results in the costs will scale with focal length and aperture size. Thus we use multiple apertures in combination with computation to break this linkage.
  - If we keep the f# and reduce the aperture size, although diffraction limit is not changed, while the object plane's resolution and FOV are reduced (for the reason that high frequency can not be included?) Thus
    - TOMBO(thin observation module by bound optics)replaces the single lens by using an array of lenses.
    - Combine diversity low resolution image from slightly different FOV to reduce the effects of detector-limited sampling, but does not improve on the diffraction-limited resolution.
  - integral imaging & light-field cameras are also example of the multi-aperture imaging, while the former one aims to reconstruct the object and the latter one is associated with sampling in k-space, which can construct the whole light-field.
- Feature-Specific and Compressive Imaging:
  - aims to measure projections of a scene onto a limited set of basis functions that encompass the set of desired features effectively as opposed to a pixel-based set of measurements to improve the SNR or finish specific task.

    image.png
    where $N_v$is the whole photons of the imager, $\sigma_n$is the fixed noise, P represents the measurement
    In addition, large arrays for imaging will cause the data volumes too large, which will increase the demands on data storage, transmission, and processing, that means the unacceptably long latencies in system operation.
  - five different feature types:
    - Hadamard: comprised by 1 & -1 with the requirement of that H * HT=NI, (thus it can project the data to a set of basis which are orthogonal to each other?) projection means that we can change a high-dimensional object to a low-dimensional object, which seems likely to the measurement of a scene so that we can solve the ill-posed problem. (最小二乘法？）
    - Haar wavelet: can be used in compression & feature extraction
      小波变换之哈尔小波 - 知乎 (zhihu.com)
      Haar 小波——原理 - 知乎 (zhihu.com)
    - Daubechies wavelets (DAUB4)：好像没怎么看懂
    - Karhunen–Loeve (KLT):
      PCA的上一层
    - independent component analysis (ICA):
      独立分量分析（Independent Component Analysis） - 知乎 (zhihu.com)
      交叉熵、相对熵（KL散度）、JS散度和Wasserstein距离（推土机距离） - 知乎

(zhihu.com)
ICA简介：独立成分分析 - 知乎 (zhihu.com)

- Compressive imaging, which is a more general method of projective imaging than feature-specfic imaging, allows one to reduce the demand for ever-increasing detector array sizes. Compressive imaging acquires and reconstructs images using projections of a scene onto a set of basis functions instead of mapping the scene onto a detector using point-to-point imaging. The minimum number of projection measurements required to reconstruct an image is determined by the sparsity of the scene and the SNR of the scene. In addition to finite extent and positivity, the scene sparsity represents additional a priori information one can apply as a constraint in post-detection processing.
  Consider a basis described by a set of vectors ψ_i,
  $$x = \sum_{i=1}^{N} \alpha_i \psi_i$$
  only K values of the coefficients are nonzero, where K<<N, compression is achieved when one stores only the K most significant coefficients. However, compression is also possible if one limits the number of measurements to M<N, where the measurements can be writtern:
  $$y = \Phi x = \Phi \Psi \alpha$$
  $$\Psi$$
  here is a set of vectors which stacks the basis, and the Φhere is a set of the measurement. In general, number of measurement will be extremely smaller than the solutions so that there will always be infinite solutions, while if we carefully design the measurement matrix, the solution can be well-approximated from the measured values. Reconstruct methods include l1 optimization:
  $$\hat{\alpha} = argmin||\alpha'||_1$$
  such that $\Phi \Psi \alpha' = y$
  $$\hat{\alpha} = argmin||\alpha'||_1$$
  such that$||y - \Phi \Psi \alpha'||_2 < \epsilon$
  In most cases, compressive sensing techniques enable a given performance to be achieved with fewer measurements than required with conventional sampling methods.

- Adaptive Systems:
  - Use SLM enables an imager to alter its PSF over time and when combined with post-detection processing, enables feedback in imaging so that we can reduce the influence of the aberration.
  - Adaptive Optics:
    - Shack-Hartmann wavefront sensor: aberration can cause the change of the point position, and we can control the MEMS device to reduce the measured aberration such as deformable mirror or SLM
  - Adaptation-Based on Information
    - PANOPTES use the concepts from information theory as a foundation for adaptation. It extracts relevant information from a scene, matches imaging resources to the information content of a scene by using micromirror technology.
  - Adaptive Lidar:
    - Conventional imaging systems require an unobscured line of sight to the object plane of interest. Range-gated obscurations is used to image through obscurations, while it is limited by the amount of illumination that penetrates the foliage to reach the object plane and returns to the transmitter.
    - Digital holography and optical phase conjugation can be used to solve this problem for the reason that they can provide a conjugate phase for the object, which make the image process success to the obscured object.

- Future of Computational Imaging:

- Strengths:
  - The features of this paradigm include altering the nature of the optical measurement to achieve a desired result (the most appropriate measurement may not result from a traditional point-to-point mapping of a scene) and balancing image formation and information extraction between the physical and computational domains.
- Weakness:
  - How to balance the costs between optics and computation.
  - Traditionally, costs are most spent on the mass-produced optics and now they are transferred to the computation.
  - If the computational model can realistically reflect the complexity of the optical system, such as the wave nature of light, aberrations, or system sensitivity to temperature variations. And the complex model may produce the time and space burden of computation.
  - Calibration is essential to Computational Imaging, which may cause more costs.
  - Joint design of optics and computation, runs counter to modular open system design, which is the trend in modern technology.
  - Recently, Computational Imaging can not replace the mature instrument such as MRI and CT unless its benefits are extremely significant.
- Opportunities:
  - Scientific and medical:
    - Scientific applications can best justify the large data processing loads and load data processing times. They take full advantage of the enhanced capabilities enabled by active imaging, adaptive optics and multi-dimensional imaging, especially spatial-spectral.
  - Commercial and industrial:
    - Commericial and industrial applications, in which the goal is to monitor human behavior or enterprise operations, have the advantage that the environment is cooperative, if not completely under user control. VR/AR is viewed as potentially profitable markets in which Computational Imaging can play a role, while these systems still need to overcome the psychophysical effects of representing a three-dimensional environment on a two dimensional display.
  - Safety, security and defense:
    - non-cooperative, non-intrusive and primarily passive, in these circumstances, spatial-spectral imagers and multi-band imagers can significantly increase capabilities.
  - Spectral bands other than visible, such as the x-ray, ultraviolet, near-IR, IR, terahertz, and gigahertz regimes.
    - As their bands need to expensive optics and lack integrated detectors, lack of a broad industrial infrastructure in these bands suggests considerable opportunity, which is no doubt an opportunity for computational imaging.
  - Both ends of the complexity-volume scale:
    - performance is the best: molecular imaging or space-based telescopes.
    - volume is limited: iPhone X, includes sophiscated illumination, image capture, and post-processing in a very small package.
  - Three-dimensional imaging, imaging through scattering media and adaptive imaging
    - Three-dimensional imaging: provide an opportunity to unique approach to privacy concerns, it can make non-pixel-based measurements to extract only information necessary for the interface to function and, thereby, preserve privacy in a fundamental way.
    - Imaging thtough scattering media: rubbish
    - Adaptive imaing:
      - i.e., an imager whose next measurement is determined based on prior measurements.
  - Threats:
    - It determines if the functionality can be realized by the conventional ways, especially for the smartphones.
    - It is likely that the emphasis in the future will shift from modifying what a camera does to simply combining images for information extraction.